Practice questions

1. Draw $\mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, together with $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$, in a single xy plane.

2. If $\mathbf{v} + \mathbf{w} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\mathbf{v} - \mathbf{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, compute and draw the vectors $\mathbf{v}$ and $\mathbf{w}$.

3. Consider the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}.$$

   (a) Compute $\mathbf{u} + \mathbf{v} + \mathbf{w}$ and $2\,\mathbf{u} + 2\,\mathbf{v} + \mathbf{w}$.

   (b) These vectors lie in a plane because $\mathbf{w} = c\,\mathbf{u} + d\,\mathbf{v}$. Find c and d.

4. Calculate the dot products $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{u} \cdot \mathbf{w}$, $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ and $\mathbf{w} \cdot \mathbf{v}$ for

$$\mathbf{u} = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

5. Compute the lengths $\| \mathbf{u} \|$, $\| \mathbf{v} \|$ and $\| \mathbf{w} \|$ of those vectors.

6. True or false (give a reason if true, or find a counterexample if false):

   (a) If $\mathbf{u} = (1, 1, 1)$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{v}$ is parallel to $\mathbf{w}$.

   (b) If $\mathbf{u}$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is perpendicular to $\mathbf{v} + 2\,\mathbf{w}$.

   (c) If $\mathbf{u}$ and $\mathbf{v}$ are perpendicular unit vectors then $\| \mathbf{u} - \mathbf{v} \| = \sqrt{2}$.

7. Calculate the vector product of $\mathbf{a}$ and $\mathbf{b}$ given that

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{b} = \mathbf{i} - \mathbf{j} - \mathbf{k}.$$

8. Calculate the vector product of $\mathbf{i} - \mathbf{j}$ and $\mathbf{i} + \mathbf{j}$.

9. Compute the cross product $\mathbf{u} \times \mathbf{v}$ and verify it is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

   (a) $\mathbf{u} = (4, 3, -2)$, $\mathbf{v} = (2, -1, -1)$

   (b) $\mathbf{u} = 2\mathbf{j} - 4\mathbf{k}$, $\mathbf{v} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

10. State whether each expression is meaningful or meaningless. Explain.

   (a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

   (b) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

   (c) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

   (d) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

   (e) $(\mathbf{u} \cdot \mathbf{v}) \times (\mathbf{w} \cdot \mathbf{x})$

   (f) $(\mathbf{u} \times \mathbf{v}) \cdot (\mathbf{w} \times \mathbf{x})$